Test1 **MTH102 Review Questions**

1. Find the indicated derivative and simplify:

   (a) $f(x) = 3x^4 - 2x^{-3} + 1$

   (b) $f(x) = \dfrac{1}{2x^2} + \dfrac{x^2}{2}$

   (c) $f(x) = (x^2 - 1)(x^3 - 3)$

   (d) $h(t) = \dfrac{2x-3}{(x+1)^2}$

   (e) $G(x) = \dfrac{1}{3x+2}$

   (f) $f(x) = (2x-3)^3$

   (g) $f(x) = 2\sqrt{x} + \dfrac{4}{\sqrt{x}}$

   (h) $y = \sqrt[3]{x^3 - 5}$

   (i) $M(x) = \left(\dfrac{2x-4}{x^2+6}\right)^{-3}$

   (j) $k(x) = \left(\dfrac{2x-4}{(2x+1)^2+6}\right)^2$

2. For the following functions find:
   - slope of the graph of the function at the given x
   - equation of the tangent line at the given x
   - the value(s) of x where the tangent line is horizontal
     1. $f(x) = x^2 + 4$, at $x = 1$
     2. $f(x) = x^4 - 32x^2 + 10$ at $x = 4$
     3. $f(x) = \dfrac{x-1}{(x-3)^3}$, at $x = 2$

3. Find each limit, if it exists:

   (a) $\lim\limits_{x\to 0} \dfrac{2x}{3x^2 - 2x}$

   (e) $\lim\limits_{x\to -3} \dfrac{x+3}{x^2+3x}$

   (b) $\lim\limits_{x\to 3} (2x^2 - x + 1)$

   (f) $\lim\limits_{x\to 0} \dfrac{x+3}{x^2+3x}$

   (c) $\lim\limits_{x\to 4^-} \dfrac{|x-4|}{x-4}$

   (d) $\lim\limits_{h\to 0} \dfrac{f(2+h) - f(2)}{h}$, $f(x) = \dfrac{1}{x+2}$

4. Use the definition of the derivative to find $f'(x)$
   a) $f(x) = x^2 - x$
   b) $f(x) = 4 + \frac{4}{x}$
   c) $f(x) = 10\sqrt{x+5}$
   d) $f(x) = \frac{3x}{x+2}$

5. Let $p = 25 - 0.01x$ and $C(x) = 2x + 9{,}000$, $0 \le x \le 2{,}500$ be the price –demand equation and the cost function respectively, for the manufacture of umbrellas.

   (A) Find the marginal cost, average cost, and marginal average cost functions.
   (B) Express the revenue in terms of x, and find the marginal revenue, average revenue, and marginal average revenue functions.
   (C) Find the profit, marginal profit, average profit, and marginal average profit functions.
   (D) Find the break-even point.
   (E) Evaluate the marginal profit at $x = 1{,}000$, $1{,}150$ and $1{,}400$, and interpret the results.
   (F) Graph $R = R(x)$ and $C = C(x)$ on the same coordinate system, and locate regions of profit and loss.

6. The price *p* (in dollars) and the demand *x* for a particular clock radio are related by the equation:

$$x=4000-40p$$

a) Express the price *p* in terms of the demand *x* and find the domain of this function.
b) Find the revenue *R(x)* from the sale of *x* clock radios. What is the domain of *R*?
c) Find the marginal revenue at a production level of 1600 clock radios and interpret?
d) Find the exact revenue from selling the 1601st clock radio? Compare your answer to that in part c)?
e) Find the average revenue from selling *x* clock radios?
f) Find the marginal average revenue if 1600 radios are sold and interpret?

7. Use the first and second derivative tests to graph the following polynomial functions:
   a) $f(x) = 1 - 3x - x^3$
   b) $f(x) = 6x(x-1)^3$